Physical Modeling of Slag Foaming for Various Operating Conditions and Slag Compositions

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The present study is concerned with steady-state slag foaming. First, the various models presented in the literature are briefly reviewed and their accuracy in predicting steady-state foam thickness is evaluated by using a wide range of experimental data. Then, a correlation is developed by applying the Buckingham's Pi theorem to variables commonly used in previous studies on slag foaming. However, the concept of foaminess has been disregarded as it is an idealization that is not supported by experimental evidences. A power type of law has been assumed between the four dimensionless numbers derived. The empirical coefficients are obtained using experimental data reported in the literature. They cover a wide range of operating conditions and foaming fluids with appreciably different chemico-physical properties. The predictions of the correlations compares well with experimental data. The dimensionless numbers derived from the Buckingham's Pi theorem and the associated empirical coefficients are found to be similar to those obtained by performing a dimensional analysis of the governing equation for the transient foam thickness which could not be predicted a priori. Finally, the dependency of the average bubble radius with surface tension is discussed. Overall, the predicted thickness decreases with increasing surface tension thus satisfying basic thermodynamics considerations.

KEY WORDS: slag foaming; physical models; bubbles; EAF; foam stability; BOF slags; gas hold-up.

1. Introduction

Slag foaming consists of introducing gas bubbles into molten metal and slag by either bubble injection or by chemical reaction. In the early days of steel manufacturing, slag foaming was viewed as a process hindering occurrence as it caused improper function of the iron extraction procedure. Therefore, studies were limited to ways of reducing or eliminating slag foaming during iron extraction and steel production particularly in oxygen steel making furnaces.[1] However, it was soon realized that slag foaming could be more beneficial in steel production. Indeed, it provides a shield for the refractory from the arc of the combustion products hence extending the life of the refractory lining.[1] In addition, slag foam prevents the melt from oxidizing and enables control of its composition.[1,2] It also acts as a thermal insulator between the hot bath and the surroundings thus reducing the electrical power required to maintain the high operating temperature and limiting electrode consumption.[3] Traditional applications include basic oxygen furnace (BOF) steelmaking, electric arc furnaces (EAF), and ladle processing where vacuum degassing is employed.[4] Recent interest was motivated by the development of innovative iron making processes, in particular bath smelting.[4] Because of the abovementioned benefits of slag foaming, it is essential to accurately predict the height of the foam in order to control the process and optimize the furnace design. Particular attention is paid to steady opera-

tion, often desired for quality assurance purposes.

2. Background

Identifying and investigating the effect of slag properties on the steady-state thickness of slag foam in iron and steel making process has been the prime objective of a series of studies by Fruehan and co-workers.[2,4–10] These studies used the concept of foaminess introduced by Bikerman.[11] It states that the foam thickness H is proportional to the superficial gas velocity j defined as the ratio of the gas flow rate to the cross sectional area of the injector pipe. The constant of proportionality Ω is the so-called "unit of foaminess" or "foaming index", i.e., $H = \Omega j$. It is a measure of the persistence of foams and corresponds to the average lifetime of a bubble in the foam before bursting.

In slag foaming experiments reported in the literature, argon, hydrogen, and helium were bubbled into molten CaO–SiO_2–FeO–MgO–Al_2O_3 slags of various compositions contained in cylindrical tanks. Ito and Fruehan[2,5] measured the steady-state foam thickness for CaO–SiO_2–FeO slags foamed with argon and performed a dimensional analysis based on the Buckingham's Pi theorem to relate the unit of foaminess Ω to the liquid viscosity μ, density ρ, and surface tension σ. Two dimensionless numbers were identified, and the foaming index Ω was found to be proportional to the ratio $\mu/\sqrt{\sigma\rho}$.[2,5] Jiang and Fruehan[6] confirmed the previous work, but suggested a different empirical constant of

proportionality between Ω and $\mu/\sqrt{\sigma\rho}$,

$$H = 115 \frac{\mu}{\sqrt{\rho\sigma}} j \quad ....(1)$$

Similarly, Skupien and Gaskel[12] use the dimensionless numbers obtained from the dimensionless analysis performed by Jiang and Fruehan.[6] They propose the following correlation for melts in the CaO–FeO–SiO$_2$ system in the temperature range from 1 573 to 1 708 K,

$$H = 100 \frac{\mu^{0.54}}{\rho^{0.39}\sigma^{0.15}} j \quad ....(2)$$

The authors also measured and investigated the effect of surface tension by varying the basicity and temperature of the melt. They found that the steady state foam height increases with decreasing surface tension and increasing viscosity. This can be attributed to the fact that the internal energy of the foam is reduced by reducing the surface tension (energy per unit surface area). A lower surface tension thus increases the stability of the foam resulting in a higher foam thickness. Finally, as the viscosity increases the liquid drains out of the foam at a slower rate. The liquid film separating the bubbles is more stable thus increasing the lifetime of bubbles in the foam as well as the foam thickness.

However, although the average bubble radius has been identified as an important parameter for the steady-state foam thickness,[13] it was not considered in the above mentioned studies. Zhang and Fruehan[8,9] recognized this point and performed the dimensional analysis using Buckingham's Pi theorem by adding the average bubble diameter D as an additional variable. Two dimensionless groups were obtained, and a power type of law was assumed to relate them. The dimensionless numbers identified were the Morton's number (Mo) and the Archimedes number (Ar) whose definitions can be found in the literature.[8] Zhang and Fruehan[8] reported that the unit of foaminess increases with viscosity and decreases with density and bubble average diameter. The final correlation obtained is written as follows,

$$H = 115 \frac{\mu^{1.2}}{D^{0.9}\rho\sigma^{0.2}} j \quad ....(3)$$

Zhang and Fruehan[8] also investigated the effect of different gases on the steady-state foam thickness. They concluded that the foam thickness decreases as the bubble radius increases. This can be attributed to the size of the bubbles formed when helium, hydrogen and argon are injected in the slag. The authors pointed out that since argon is much denser than both helium and hydrogen, the size of the bubbles generated by injection of the argon is greater than the latter gases injected at similar volumetric gas flow rates. This is due to the momentum of gases and buoyancy forces required to overcome the surface tension of the liquid.

The correlations presented by Fruehan and co-workers were developed by using a limited range of chemico-physical properties and operating conditions. Consequently, it may not predict the steady-state thicknesses of foams generated from fluids with appreciably different physical properties as first recognized by Ghag et al.[14] Moreover, the above mentioned studies were based on the concept of foaminess which is assumed to be an intrinsic property of the foaming solution, independent of the apparatus and of the procedure used. In practice, however, "these ideals have not been achieved"[15] and the concept of foaminess is questionable. Moreover, slag starts foaming beyond a minimum superficial gas velocity j_m.[16,17] This is not accounted for by the foaming index. This was also questioned by Lin and Guthrie.[13] Pilon et al.[16] also addressed this issue in their study of steady state thickness of liquid-gas foams.

According to Ghag et al.,[18,19] based on experimental measurements, rupture takes place in slag films at thicknesses when the main stabilizing forces are viscoelastic rather than thin film interacting forces. Thin film interacting forces arise due to the overlapping of force fields of two interfaces in proximity of each other. On the other hand viscoelastic forces are present in well drained spherical bubbles. At room temperature, bubbles stabilized by viscoelastic forces can be achieved by limiting the bubbles to spherical form.[18,19] In doing so, the experimental difficulties associated with high temperature laboratory scale foaming experiments are eliminated. Following this argument, Ghag et al.[18,19] modeled high temperature slag foaming by injecting compressed air into a binary distilled water-AR grade glycerol solutions as base solutions to provide a range of viscosities and sodium dodecylbenzene sulphonate (SDBS) to control the surface tension. Based on their experimental results the authors concluded that viscosity has a larger influence than surface tension on the foam height. In the development of a model for slag foaming, Ghag et al.[18] pointed the poor agreement between the predictions of Eq. (3) and their experimental data. Instead, they suggested the following correlation[18]

$$H = 5 \times 10^5 \frac{\mu E_{eff}}{(\rho g D)^2} \quad ....(4)$$

where, E_{eff} is the effective elasticity of the bubble film. The model developed by Ghag et al.[18] shows good agreement for their data. The main drawback in using Eq. (4) is that the determination of the E_{eff} is very tedious and requires additional parameters, which were not available for the set of data used in this study and therefore Eq. (4) could not be used. Moreover, even though it was claimed that Eq. (4) can be applied to high temperature slag foaming, Ghag et al.[14,18,19] did not compare their model with data reported previously in the literature for other slag foam or model solutions.

Zhu and Du Sichen[20] performed experimental investigation by injecting nitrogen into three types of silicon oil of different viscosity and surface tension. The authors identified two foaming mechanisms namely one layer and two layers foaming. In the one layer foaming mechanism, which is described as the most common one, the authors found the foam filled the whole container and the final foam height was twice the original slag height. An upper region with a high gas fraction and a lower layer with lower gas content characterize the two-layer mechanism. The authors claim that the formation of the two layers depends more on the surface tension of the slag than on the viscosity. The authors developed a semi-empirical mathematical model to predict foam thickness based on continuity and momentum conservation equations for the gas phase in the vertical direction. Application of the model developed requires the

determination of bubble Reynolds number as proposed by Kuo and Wallis[21] and of the drag coefficient. However, the expression presented for the calculation of the Reynolds number does not give a dimensionless number. As a result, the expression presented by the authors leads to predicted foam heights with inconsistent units and thus the application of the proposed correlation appears to be erroneous. The authors also presented a semi-empirical formula for the bubble diameter, which is a combination of the viscosity to density ratio and surface tension and density ratio, as proposed by Davidson and Schuler[22] and Sano and Mori,[23] respectively. Once again, the final expression presented by the authors is, however, not homogeneous and therefore could not be used in this study.

More recently, Pilon and co-workers[16,17] derived a model to predict the steady-state thickness of pneumatic foam based on the expressions for the drainage equations available in the literature.[24–28] The expressions were combined to obtain the time rate of change of the foam thickness. A dimensionless analysis of the area-averaged drainage equation was performed and two dimensionless numbers Re/Fr and Ca×H/r were obtained, where the Reynolds, Froude, and Capillary numbers are defined respectively as,

$$\text{Re}=\frac{\rho(j-j_\mathrm{m})r}{\mu}, \quad \text{Fr}=\frac{(j-j_\mathrm{m})^2}{gr},$$

$$\text{and} \quad \text{Ca}=\frac{\mu(j-j_\mathrm{m})}{\sigma} \quad \dots\dots\dots\dots(5)$$

where, ρ is the density of the liquid, j is the superficial gas velocity, j_m is the minimum superficial gas velocity required for foaming, μ is the viscosity, σ is the surface tension, r is the average bubble radius at the bottom of the foam layer, and g is specific gravity. The Reynolds number represents the ratio of the inertial and the viscous forces, the Capillary number is the ratio of the viscous and the surface tension force, and the Froude number the ratio of the inertial and the gravitational forces acting on the bubbles. The final correlation was obtained by assuming a power law relationship whose empirical constants were found from a wide variety of experimental data for high viscosity fluids and leading to

$$H=2\,905\frac{\sigma[\mu(j-j_\mathrm{m})]^{0.8}}{(\rho g)^{1.8}\,r^{2.6}} \quad \dots\dots\dots\dots(6)$$

The drawback in using Eq. (6) is that one needs to find the minimum superficial gas velocity j_m in order to predict the steady state foam thickness from the chemico-physical properties of the liquid/gas system. To address this issue, a self-contained semi-empirical model for j_m has been suggested by Pilon and Viskanta.[17]

Finally, Eq. (6) appears to be in contradiction with thermodynamic principles previously discussed. Also, assuming that surface tension and bubble radius are independent variables, the foam thickness H should be proportional to the bubbles radius r and inversely proportional to surface tension σ. This was attributed to the fact that the bubble radius and surface tension are not independent variables.

Figure 1 compares the steady-state foam thickness predicted by the diverse correlations reporter in the literature and that measured experimentally and summarized in Ref.
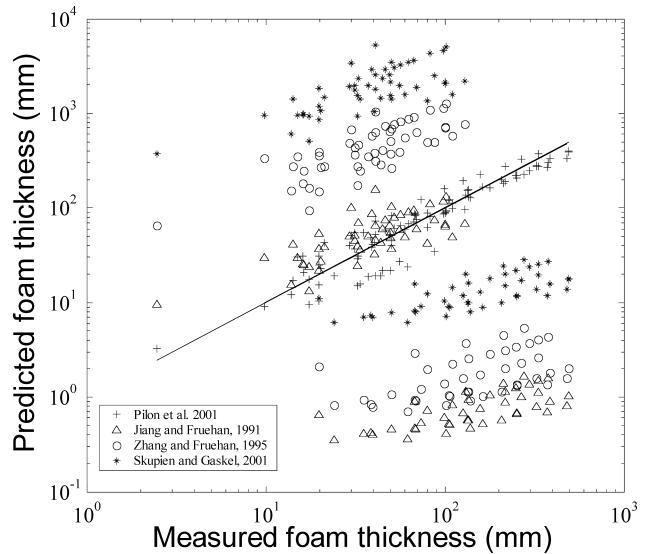
Fig. 1. Comparison between experimental data and predictions of steady-state slag foam thickness obtained from previously reported correlations. The solid line represents the exact predictions.

24). The solid line represents the expected results. It can be noted that there is a marked discrepancy between the various correlations and experimental data. The discrepancies reflected in Fig. 1 can be attributed to the fact that the empirical coefficients in Eqs. (1) to (3) were determined from a limited set of experimental data and used the concept of foaminess whose limitation have been described earlier. On the other hand, the correlation presented by Pilon *et al.*[16] most accurately predicts, albeit within 35%, the foam thickness over a wider range of experimental data. The present paper disregards the concept of foaminess but applies the Buckingham's Pi theorem to basic variables describing the foam and its constituents. It also addresses the apparent contradiction due to presence of the surface tension in the numerator of Eq. (6).

3. Analysis and Results

In the slag foaming literature it is common practice to perform dimensionless analysis based on the Buckingham's Pi theorem to obtain correlations between foaminess and the liquid properties. It is a very practical and robust theorem, however, one should be careful when applying the Buckingham's Pi theorem, as there is no rigorous and systematic rule to which variables should be chosen as repeating variables.[29] Dimensionless analyses may not always lead to a physically sound set of dimensionless numbers that field equations and boundary conditions can provide.

A priori, the steady state foam thickness H, depends on six variables: the liquid viscosity μ, the surface tension σ, the average bubble radius r, the liquid density ρ, the specific gravity g, and the reduced superficial gas velocity $j_\mathrm{r}=j-j_\mathrm{m}$. The reduced superficial gas velocity j_r is used instead of the superficial gas velocity j because foam does not occur for j less than j_m and j_m can be defined in terms of liquid properties.[17] The minimum superficial gas velocity j_m was estimated using the analytical expression suggested by Pilon and Viskanta.[17] By disregarding the concept of

foaminess the Buckingham's Pi theorem is applied to the above-mentioned variables. The seven variables consist of a combination of the three basic physical dimensions for length [L], mass [M] and time [T]. According to the Buckingham's Pi theorem, a function relating these seven variables can be expressed in terms of four dimensionless parameters. Choosing $r^{a_1} \mu^{b_1} j_r^{c_1} H$, $r^{a_2} \mu^{b_2} j_r^{c_2} \sigma$, $r^{a_3} \mu^{b_3} j_r^{c_3} \rho$ and $r^{a_4} \mu^{b_4} j_r^{c_4} g$, these combinations become dimensionless when $a_1=b_2=b_3=c_2=-1$, $a_2=b_1=b_4=c_1=0$, $a_3=a_4=c_3=1$ and $c_4=-2$. The dimensionless numbers obtained are as follows,

$$\Pi_1 = \frac{H}{r}, \quad \Pi_2 = \frac{\sigma}{j_r \mu} = \frac{1}{\mathrm{Ca}},$$

$$\Pi_3 = \frac{gr}{j_r^2} = \frac{1}{\mathrm{Fr}}, \quad \text{and} \quad \Pi_4 = \frac{\rho r j_r}{\mu} = \mathrm{Re} \quad \dots\dots\dots(7)$$

where, Π_2 and Π_3 are the reciprocal of the Capillary and Froude numbers while Π_4 is the Reynolds numbers defined in Eq. (5). Interestingly, a combination of these dimensionless was also obtained by Pilon et al.[16]

The solutions for the set of dimensionless numbers has the form $H/r = g(\mathrm{Ca}, \mathrm{Re}, \mathrm{Fr})$, respectively. Assuming a power law relationship one assumes,

$$\frac{H}{r} = k\mathrm{Ca}^{\alpha}\mathrm{Re}^{\beta}\mathrm{Fr}^{\gamma} \quad \dots\dots\dots\dots\dots(8)$$

The constants k and the exponents α, β, and γ in Eq. (8) were obtained from experimental data gathered from various sources and summarized by Pilon et al.[16] The values of the coefficient and the exponent was found to be as follows, $k=2617\pm1$, $\alpha=-1.01\pm0.02$, $\beta=-1.74\pm0.02$ and $\gamma=1.77\pm0.04$ with a regression coefficient R^2 of 0.98. The final expression can therefore be expressed in terms of the dimensionless numbers or the chemico-physical properties, respectively, as

$$\frac{H}{r} = 2617\mathrm{Ca}^{-1.01}\mathrm{Re}^{-1.74}\mathrm{Fr}^{1.77} \quad \dots\dots(9)$$

and

$$\frac{H}{r} = 2617 \frac{\mu^{0.73} j_r^{0.79} \sigma^{1.01}}{\rho^{1.74} g^{1.77} r^{3.51}} \quad \dots\dots\dots(10)$$

Figure 2 shows a comparison of the predicted steady-state foam thickness predicted by the Eq. (10) with the measured foam thickness. It can be seen that the foam thickness predicted by both correlations agrees well (approximately within 35%) with measured foam thickness. The regression coefficient, R^2 in this case was found to be 0.93. This shows good agreement between the predicted and measured foam thicknesses and is comparable to the accuracy of the model presented by Pilon et al.[16]

4. Discussion

First of all is it worth noting that the expressions obtained in Eqs. (9) and (10) are in fact similar to that presented by Pilon et al.[16] with exponent approximately 10% close to each other and could not have been predicted a priori. It should be underlined that, Pilon et al.[16] obtained only two independent dimensionless numbers by using field
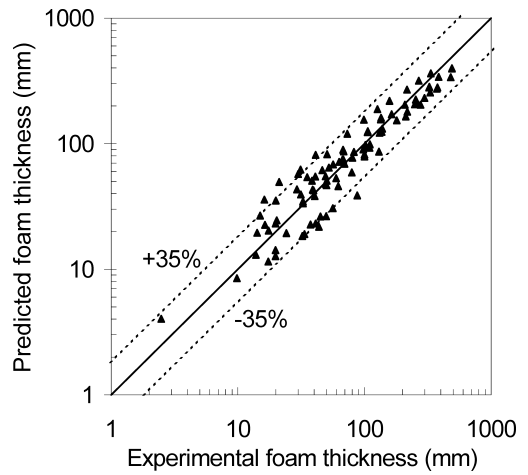
Fig. 2. Comparison between experimental data and slag foam thickness predicted by Eq. (10).

equations and the boundary conditions. These numbers can be expressed as a combination of the four dimensionless numbers independently found in this study. Therefore, it can be safely stated that the dimensionless numbers obtained in this study are in fact physically meaningful.

As mentioned earlier, even though the agreement between the experimental and the foam thickness predicted by Eq. (6) or (10) are good, the presence of surface tension in the numerator instead of the denominator seems to contradict intuition based on thermodynamics principle. To address this issue, the dependence of the bubble radius on the chemico-physical properties of the system and particularly the surface tension should be discussed along with the physical mechanism responsible for bubble formation. The bubble radius forming the foam depends on numerous factors such as the method by which the bubbles are formed and the chemico-physical properties of the foaming liquid. If bubbles are formed by chemical reaction within the liquid, they tend to be small, spherical, and monodispersed. On the other hand, if bubbles are generated by injection through an orifice or a perforated surface, the aperture at the formation site will also dictate their size. The bubble growth and detachment mechanisms from the formation site have been reported in numerous studies.[11,30] At the moment of detachment the main forces acting on the bubble are the buoyancy forces, the added mass force, and the surface tension as reported by Loubière et al.[30] The authors also report that the other forces acting due to the momentum of the gas and the viscous drag are negligible at the time of detachment based on experimental investigation. Bickerman[11] presented an explanation and theory of the bubble growth and detachment dynamics for bubbles at low and high injection rate. A detailed and in depth discussion of the complex dynamics of bubble formation falls beyond the scope of this study.

However, the dependence of the bubble radius on the ratio of density and surface tension ratio reported by Sano and Mori[23] is worth considering in order to understand the presence of the surface tension in the numerator of Eqs. (6) and (10). The authors reported that the average bubble size is proportional to $(\sigma/\rho)^{0.5}$ where σ is the surface tension and ρ is the density of the liquid, respectively. **Figure 3** shows the bubble radius as a function of $(\sigma/\rho)^{0.5}$. The den-
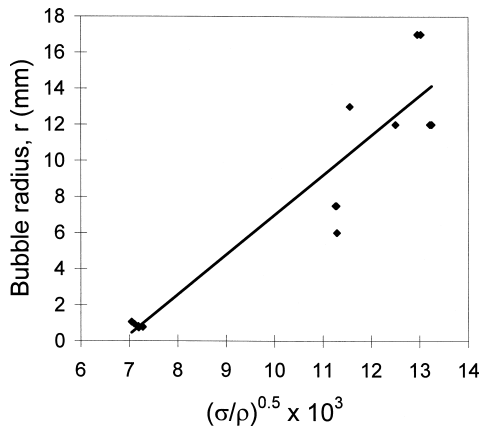
Fig. 3. Variation of the average bubble radius as a function of $(\sigma/\rho)^{0.5}$.
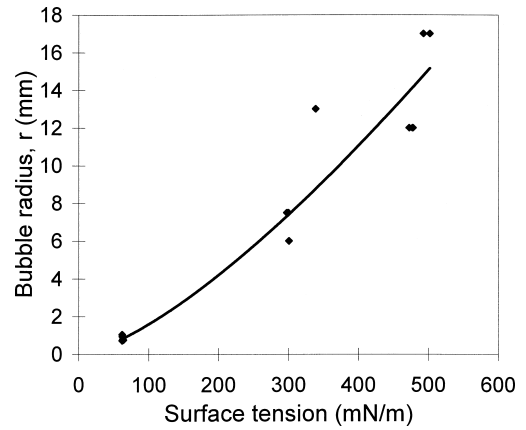

Fig. 4. Variation of the average bubble radius with surface tension.

Table 1. Summary of thickness of Experimental Data (reproduced from Ref. 16)).

Solution	Dimensions i.d. & H_o	Gas	Nozzle type	Gas flux (mm/s)	σ (mN/m)	μ (mPa.s)	ρ (kg/m³)	T (°C)	r_o (mm)	Ref.
40% CaO-40%SiO₂- 5%FeO- 10%Al₂O₃	i.d. = 9.2 cm H_o = 4.5 cm	Argon	Single and Multiple	0 to 50	463	398	2743	1500	7.8 to 13.5	Zhang and Freuhan[8]
48%CaO-32%SiO₂- 10%FeO-10%Al₂O₃	i.d.= 4.1 cm H_o = 4.2 cm	Argon	Single	0 to 30	477.2	381	2733	1600	12	Ozturk and Freuhan[10]
75SiO₂-15NaO₂- 10CaO(wt%)glass	i.d. = 6.5 cm H_o = 2.0 cm	Air	Single	0 to 2.5	297.7 to 307.7	7450 to 12100	2346 to 2358	1425 to 1500	15 to 20	Laimbock[31]
Water + 78% to 95% glycerinate +SDBS	i.d. = 10.7 cm H_o = 16.7 cm	N₂	Pyrex disk	0.83 to 1.5	69.5 to72.3	46.5 to 520.8	1204 to 1251	20	0.7 to 1.1	Ghag et al.[16]
30% FeO-42%SiO₂- 28%CaO	i.d. = 3.2/5cm H_o = N.A	Argon	Single	0 to 27.0	477.9	1605	3055	1300	12	Ito and Freuhan[2]
3%FeO (Ca/SiO₂ = 1.25)	i.d. = 9.2 cm H_o = 4.5 cm	Argon	Single	0 to 30.3	477.2	381	2733	1500	12	Jiang and Freuhan[6]
0% FeO (Ca/SiO₂ = 1.25)	i.d. = 9.2 cm H_o = 4.5 cm	Argon	Single	0 to 40.4	472.8	396	2693	1500	12	Jiang and Freunhan[6]
30%CaO- 60%SiO₂- 10%CaF₂	i.d. = 4.1 cm H_o = 4.5 cm	Argon, He,H₂	Single	0 to 40.4	338	533	2534	1400 or 1500	13	Zhang and Freunhan[8,9]
34.78%CaO-33.76%SiO₂- 22.52%FeO- 8.94%MgO	i.d. = 4.5 cm H_o = 4 cm	Argon	Single	0 to 67.3	502	270	2956	1600	17	Jung and Freuhan[7]
37.39%CaO-35.57%SiO₂- 20.87%FeO- 6.17%Al₂O₃	i.d. = 4.5 cm H_o = 4 cm	Argon	Single	0 to 67.4	493	291	2936	1600	17	Jung and Freuhan[7]

sity varied from 1 250 to 3 000 kg/m³ while the surface tension varied from 62 to 500 mN/m. The experimental data used in this study agree with Sano and Mori's[23] claims' and the constant of proportionality equals 0.41 with a regression coefficient R^2 of 0.91.

Moreover, **Figure 4** shows the variation of the average bubble radius with surface tension. The power law curve fit indicates that the radius r varies as $\sigma^{1.4}$ with a regression coefficient of 0.98. Next if the surface tension in the numerator of Eqs. (6) and (10) are considered together with the presence of the bubble radius in the denominator it can be seen that the overall effect of an increase in surface tension results in a reduction in the predicted foam thickness. Therefore, the predictions by the correlations do not contradict the fact that foam stability increases as surface tension decreases. However, the bubble radius cannot be written solely in terms of the liquid properties since it is also dictated by the formation mechanism.

5. Conclusions

The present study has been concerned with steady-state slag foaming. First, the various models presented in literature were briefly reviewed and their accuracy in predicting

steady state foam thickness was evaluated by using a wide range of experimental data. The model presented by Pilon et al.[16] was found to give the most satisfactory results. However, the model for the foam thickness [Eq. (6)] contradicts basic thermodynamics considerations as it predicts an increase in the foam thickness as surface tension increases.

A correlation has been developed by applying the Buckingham's Pi theorem to variables commonly used in slag foaming analysis. The concept of foaminess has been disregarded as it is an idealization that is not supported by experimental evidences. The two dimensionless numbers obtained by Pilon et al.[16] by performing a dimensional analysis of the transient governing equation for the foam thickness can be expressed as functions of the four dimensionless numbers derived from the Buckingham's Pi theorem. A power type of law has been assumed between the these four independent dimensionless numbers. The empirical coefficients were obtained using experimental data for foams made of various fluids with appreciably different chemico-physical properties. The predictions of the correlations compares well with the experimental data for high viscosity liquids.

Finally, the average bubble radius was shown to increase with surface tension. Overall, the predicted thickness de-

creases with increasing surface tension thus satisfying basic thermodynamics considerations.

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